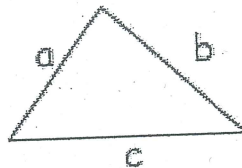
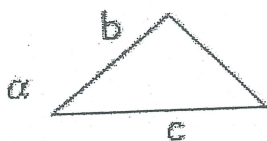


12.3 – Law of Cosines

*Used to solve oblique triangles, when you do not know an angle and its opposite side
(In other words, when you cannot use Law of Sines.)

- 1) Two sides and an included angle: SAS (1st: Find side opposite given angle; 2nd: Find angle opposite shortest side – it will always be acute.)
- 2) Three sides: SSS (1st: Find angle opposite longest side – this will take care of obtuse angle, if present; 2nd: Find either remaining two sides, they must be acute.)

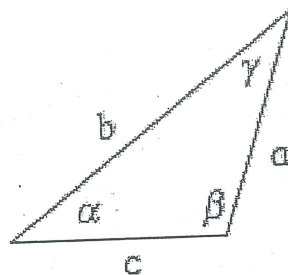


Law of Cosines

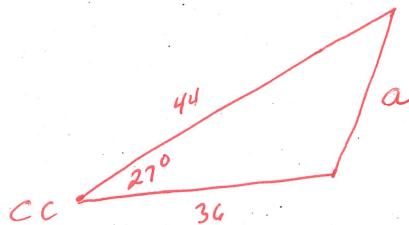
$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$



Example 1: Two tankers leave Corpus Christi, Texas, with an angle of 27° between their paths. After 2 hours, the first tanker is 44 miles from Corpus Christi and the second tanker is 36 miles from Corpus Christi. What is the distance between the two tankers?



$$a^2 = b^2 + c^2 - 2bc \cos A$$

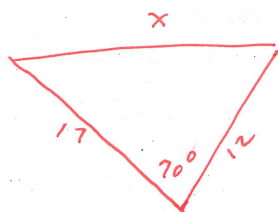
$$a^2 = 44^2 + 36^2 - 2(44)(36) \cos 27^\circ$$

$$a^2 = 1936 + 1296 - 3168 \cos 27^\circ$$

$$\sqrt{a^2} = \sqrt{409.2913314}$$

$$a \approx 20.2 \text{ miles}$$

Example 2: Two hot-air balloons approach a landing field. One is 12 meters from the landing point and the other is 17 meters from the landing point. The angle between the balloons is 70° . How far apart are the two balloons?



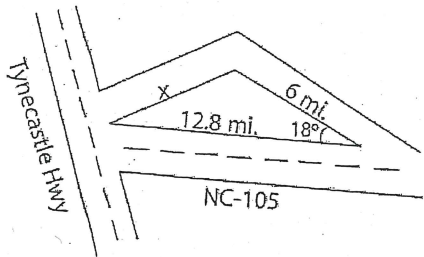
$$x^2 = 17^2 + 12^2 - 2 \cdot 17 \cdot 12 \cos 70^\circ$$

$$x^2 = 289 + 144 - 408 \cos 70^\circ$$

$$x^2 = 293.4557815$$

$$x \approx 17.1 \text{ meters}$$

Example 3: You are heading to Beech Mountain for a ski trip. Unfortunately, state road 105 in North Carolina is blocked off due to a chemical spill. You have to get to Tynecastle Highway, which leads to the resort at which you are staying. NC-105 would get you to Tynecastle Highway in 12.8 miles. The detour begins with a 18° veer off onto a roads that runs through the local city. After 6 miles, there is another turn that leads to Tynecastle Hwy. Assuming that both roads on the detour are straight, how many extra miles are you traveling to reach your destination?



$$x^2 = 6^2 + 12.8^2 - 2(6)(12.8)\cos 18^\circ$$

$$x^2 = 36 + 163.84 - 153.6\cos 18^\circ$$

$$x^2 = 53.7577191$$

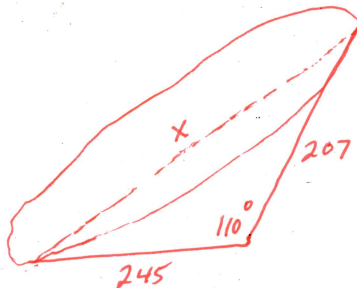
$$x = 7.3$$

$$\text{Total} = 7.3 + 6 = 13.3$$

$$- 12.8$$

$$\boxed{.5 \text{ miles}}$$

Example 4: To approximate the length of a lake, a surveyor starts at one end of the lake and walks 245 yards. He then turns 110° and walks 207 yards until he arrives at the other end of the lake. Approximately how long is the lake?



$$x^2 = 245^2 + 207^2 - 2(245)(207)\cos 110^\circ$$

$$x^2 = 60,025 + 42,849 - 101,430\cos 110^\circ$$

$$x^2 = 137,565,1031$$

$$x = 370.897699$$

$$\boxed{x \approx 370.9 \text{ yards}}$$

Example 5: A triangular plot of land has sides $a = 217$ ft, $b = 362$ ft, and $c = 345$ ft. Find the measures of all three angles.

$$\textcircled{1} \quad 362^2 = 217^2 + 345^2 - 2 \cdot 217 \cdot 345 \cos B$$

$$131,044 = 47,089 + 119,025 - 2 \cdot 74,865 \cos B$$

$$131,044 = 166,114 - 149,730 \cos B$$

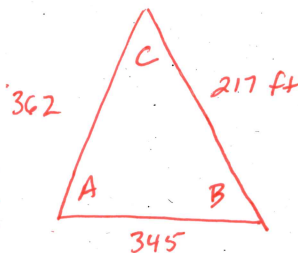
$$\begin{aligned} -166,114 & \quad -166,114 \\ -35,070 & = -149,730 \cos B \\ \frac{-35,070}{-149,730} & = \frac{-149,730 \cos B}{-149,730} \end{aligned}$$

$$.23422 = \cos B$$

$$B = \cos^{-1}(.2342215989)$$

$$\boxed{B = 76.5^\circ} \quad 76.45425685$$

$$\textcircled{2} \quad \frac{\sin 76.454}{362} = \frac{\sin C}{345}$$



$$\frac{362 \sin C}{362} = \frac{345 \sin 76.454}{362}$$

$$\sin C = .926528$$

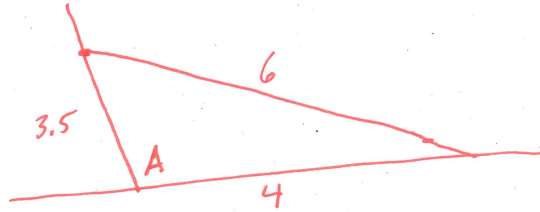
$$\boxed{C = 67.9^\circ}$$

$$A = 180 - (67.9 + 76.5)$$

$$= 180 - 144.4$$

$$\boxed{A = 35.6^\circ}$$

Example 6: After a hurricane, the small tree in my neighbor's yard was leaning. To keep it from falling, we nailed a 6-foot strap into the ground 4 feet from the base of the tree. We attached the strap to the tree 3.5 feet above the ground. At what angle is the tree leaning?



$$6^2 = 3.5^2 + 4^2 - 2(3.5)(4)\cos A$$

$$36 = 12.25 + 16 - 28\cos A$$

$$36 = 28.25 - 28\cos A$$

$$\frac{7.75}{-28} = \frac{-28\cos A}{-28}$$

$$-.2767857 = \cos A$$

$$A = \cos^{-1}(-.2767857)$$

$$A = 106.1^\circ$$